1970 AB3/BC2

$f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{4}{3}x^{-\frac{2}{3}} = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

factor $x^{\frac{1}{3}} + x^{-\frac{2}{3}} = x^{-\frac{2}{3}}\left(\boxed{x}^{A} + \boxed{1}^{B}\right)$

$\frac{x^{-\frac{2}{3}}}{x^{-\frac{2}{3}}}$

$x^{-\frac{2}{3}} \cdot \boxed{}^{A} = x^{\frac{1}{3}}$

$\boxed{}^{A} = \frac{x^{\frac{1}{3}}}{x^{-\frac{2}{3}}} = x^{\frac{1}{3}-\left(-\frac{2}{3}\right)} = x$

factor $x^{\frac{1}{3}} + x^{-\frac{2}{3}} = x^{\frac{1}{3}}\left(1 + \frac{1}{x}\right)$

$\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$ $\quad \frac{x^{-\frac{2}{3}}}{x^{\frac{1}{3}}} = x^{-1}$

factor $x^{\frac{1}{3}} + x^{-\frac{2}{3}}$

$= 5\left(\frac{x^{\frac{1}{3}}}{5} + \frac{x^{-\frac{2}{3}}}{5}\right)$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{4}{3}x^{-\frac{2}{3}} = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

CRITICAL POINTS

possible extrema x-values

f' und $\quad$ $x = 0$

$f' = 0$ $\quad$ $x + 1 = 0$ or $x = -1$

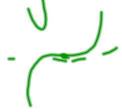
a) horizontal tangent i.e. slope = 0

$x = -1$

$y = x^{\frac{4}{3}} + 4x^{\frac{1}{3}} = 1 - 4 = -3$

$(-1, -3)$

think $y = \sqrt[3]{x}$



b) vertical tangent

slope is undefined because denominator is 0 and numerator is not.

$x = 0$

$y = 0$

$(0, 0)$

Jan 24-7:19 AM

c) $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$ on $[-8, 8]$

$f'(x) = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

crit #s at $x=0, x=-1$

Either an absolute extremum is at an end point of the interval (or curve)

OR it is in the middle [[interior]] of the interval

If it is.... it will also be a relative extremum

sign chart of $f'(x) = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

rel min

$- - - - - \mid + + + \mid + + + +$

$-8 \quad -4 \quad -1 \quad -\frac{1}{2} \quad 0 \quad 1 \quad 8$

$(-4)^{-\frac{2}{3}} = \frac{1}{(-4)^{2/3}} = \frac{1}{\sqrt[3]{16}}$ $\quad (-\frac{1}{2})^{-\frac{2}{3}}$ is pos $\quad$ pos

$(x+1)$ when $x=-4$ is neg $\quad (-\frac{1}{2}+1)$ is pos $\quad$ pos

check

$f(-8) = (-8)^{\frac{4}{3}} + 4(-8)^{\frac{1}{3}}$

$= (-2)^4 + 4(-2)$

$= 16 - 8 = 8$

$f(-1) = (-1)^{\frac{4}{3}} + 4(-1)^{\frac{1}{3}}$

$= (-1)^4 + 4(-1) = -3$

$f(8) = 8^{\frac{4}{3}} + 4(8)^{\frac{1}{3}}$

$= 16 + 8 = 24$

$f(0) = 0$

Jan 24-7:20 AM

d) $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$ on $[-8, 8]$

$f'(x) = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

crit #s at $x=0$, $x=-1$

$f(8) = 24$
$f(0) = 0$
$f(-1) = -3$
$f(-8) = 8$

concavity is a 2^{ND} derivative property

$$f''(x) = \frac{4}{9}x^{-\frac{2}{3}} + -\frac{8}{9}x^{-\frac{5}{3}} = \frac{4}{9}x^{-\frac{5}{3}}(x-2)$$

OR $f''(x) = -\frac{8}{9}x^{-\frac{5}{3}}(x+1) + \frac{4}{3}x^{-\frac{2}{3}}(1)$

Possible points of inflection	f'' und	$f''=0$
	$x=0$	$x-2=0 \Rightarrow x=2$

sign chart of $f''(x) = \frac{4}{9}x^{-\frac{5}{3}}(x-2)$

-8	-1	0	1	2	π	8
	$++++$		$- - - -$		$++++++$	

c-dn: $(0, 2)$

e)

$f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$ on $[-8, 8]$

$f'(x) = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

crit #s at $x=0$, $x=-1$

$f(8) = 24$

$f(0) = 0$

$f(-1) = -3$

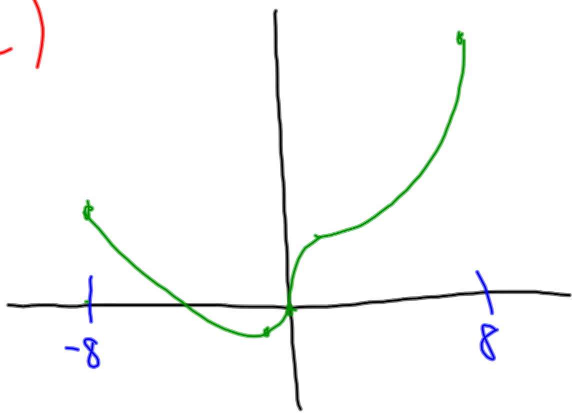
$f(-8) = 8$

c-dn: $(0, 2)$

$f(2) = (2^{\frac{1}{3}})^4 + 4(2^{\frac{1}{3}})$

$= 2(2^{\frac{1}{3}}) + 4(2^{\frac{1}{3}}) = 6 \cdot 2^{\frac{1}{3}}$

$(2^{\frac{1}{3}})^4 = (2^{\frac{1}{3}})^3(2^{\frac{1}{3}})^1$



WINPLOT